LECTURER5

DERIVATIVE RULES

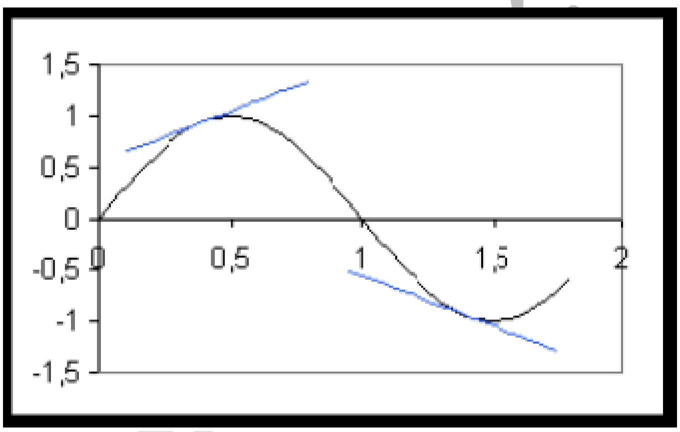
Definition

The derivative of a function f at a point x, written $f'(x)$, is given by:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

if this limit exists.

Graphically, the derivative of a function corresponds to **the slope of its tangent line at one specific point.** The following illustration allows us to visualise the tangent line (in blue) of a given function at two distinct points. Note that the slope of the tangent line varies from one point to the next. The value of the derivative of a function therefore depends on the point in which we decide to evaluate it. By abuse of language, we often speak of the slope of the function instead of the slope of its tangent line.



Notation

Here, we represent the derivative of a function by a prime symbol. For example, writing $f'(x)$ represents the derivative of the function f evaluated at point x. Similarly, writing $(3x + 2)'$ indicates we are carrying out the derivative of the function $3x + 2$. The prime symbol disappears as soon as the derivative has been calculated.

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$